Mark scheme for Extension Worksheet – Topic 2, Worksheet 6

1 **a** The gravitational field strength is the slope of the potential–distance graph; the slope is zero when $\frac{r}{d} = 0.80$; and so

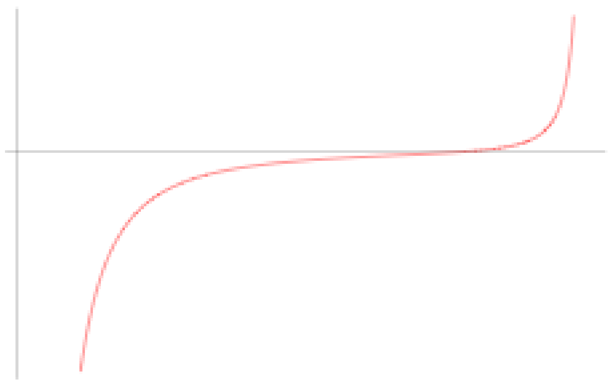
$r = 0.80 \times d = 0.80 \times 4.0 \times 10^6 = 3.2 \times 10^6$ m [3]

b $\frac{GM}{(3.2 \times 10^6)^2} = \frac{Gm}{(0.8 \times 10^6)^2}$; hence $\frac{M}{m} = \left(\frac{3.2}{0.8}\right)^2 = 16$ [2]

c The work done is $W = m\Delta V$ and so $W = m\Delta V = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\Delta V}$; i.e.

$v = \sqrt{2 \times (-0.15 \times 10^8 - (-0.30 \times 10^8))} = 5.5 \times 10^3 \text{ m s}^{-1}$ [2]

d See graph below: negative values indicate a gravitational field strength directed to the left.



[2]

2 **a** The force on m from one of the big masses is $F = \frac{GMm}{r^2} = \frac{GMm}{a^2 + x^2}$; hence the net force is $F_{net} = 2 \times \frac{GMm}{a^2 + x^2} \times \cos\theta$; which is

$F_{net} = 2 \times \frac{GMm}{a^2 + x^2} \times \frac{x}{\sqrt{a^2 + x^2}} = \frac{2GMm}{(a^2 + x^2)^{3/2}} x$, directed towards the position of equilibrium. [3]

b If x is small compared to a then we may neglect x in the denominator to get approximately, $F_{net} \approx \frac{2GMm}{a^3} x$; the acceleration is therefore $a \approx -\frac{2GM}{a^3} x$ is opposite to the displacement and proportional to it so SHM oscillation will take place. [2]

c $\omega^2 = \frac{2GM}{a^3} \Rightarrow \omega = \sqrt{\frac{2GM}{a^3}}$; and so $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{a^3}{2GM}}$ [2]

3 **a** To find the escape speed: $\frac{1}{2}mv_{esc}^2 + mV_0 = E_T$ is the total energy of the probe at launch; to just escape this has to be zero; hence $v_{esc} = \sqrt{-2V_0}$. [2]

b $v_{esc} = \sqrt{-2V_0} = \sqrt{2 \times 2.8 \times 10^9} = 7.5 \times 10^4\ \mathrm{m\,s^{-1}}$ [1]

c The total energy at launch is $\frac{1}{2}mv^2 + mV_0 = \frac{1}{2}m\frac{1}{4}(-2V_0) + mV_0 = \frac{3}{4}mV_0$; the total energy at the top point is $mV = -\frac{GMm}{r} = -\frac{GMm}{R}\frac{R}{r} = mV_0\frac{R}{r}$; hence

$\frac{3}{4}mV_0 = mV_0\frac{R}{r} \Rightarrow r = \frac{4}{3}R \Rightarrow h = \frac{R}{3} = 1.5 \times 10^5\ \mathrm{m}$ [3]

4 **a** From $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r}$; hence the total energy is

$E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}m\frac{GM}{r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ [2]

b The total energy at launch is $E_T = \frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R} - \frac{GMm}{R} = -\frac{GMm}{4R}$;

hence $-\frac{GMm}{2r} = -\frac{GMm}{4R} \Rightarrow r = 2R$ [2]

c The frictional force will reduce the total energy of the probe since some thermal energy will have to be produced; from $E_T = -\frac{GMm}{2r}$ it follows that (i) orbit radius must become smaller; and from $v^2 = \frac{GM}{r}$ it follows that (ii) the speed will increase. [3]